Completing the Square when the Coefficient of x^2 is Not Equal to One

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Completing the Square

When quadratic equations are expressed in the standard form $ax^2 + bx + c = 0$, many equations have a coefficient of x^2 that is not equal to one $(a \neq 1)$. Consider the example below:

$$2x^2 + 6x - 7 = 0$$

$$2x^2 + 6x = 7$$

Suppose that you attempt to complete the square by adding 9 to both sides of the equation.

$$2x^2 + 6x + \mathbf{9} = 7 + \mathbf{9}$$

$$2x^2 + 6x + 9 = 16$$

~~$(x + 3)(x + 3) = 16$~~

~~$(x + 3)^2 = 16$~~

Adding 9 does not complete the square. $2x^2 + 6x + 9$ is *not* equal to $(x + 3)(x + 3)$. The coefficient of 2 cannot simply be ignored. So before you attempt to complete the square it is helpful to transform the equation such that the coefficient of x^2 becomes one. Here is how it is done:

$$2x^2 + 6x - 7 = 0$$

$$2x^2 + 6x = 7$$

Multiply both sides by $\frac{1}{2}$ such that the coefficient on x^2 becomes one.

$$\frac{1}{2} \cdot (2x^2 + 6x) = \frac{1}{2} \cdot (7)$$

$$x^2 + 3x = \frac{7}{2}$$

Now you can add a constant to both sides to complete the square.

$$x^2 + 3x + \mathbf{\frac{9}{4}} = \frac{7}{2} + \mathbf{\frac{9}{4}}$$

$$\left(x + \frac{3}{2}\right)\left(x + \frac{3}{2}\right) = \frac{14}{4} + \frac{9}{4}$$

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$$\left(x+\frac{3}{2}\right)^2=\frac{23}{4}$$

$$x+\frac{3}{2}=\pm\sqrt{\frac{23}{4}}$$

$$x=-\frac{3}{2}\pm\frac{\sqrt{23}}{\sqrt{4}}$$

$$x=-\frac{3}{2}\pm\frac{\sqrt{23}}{2}$$

$$x=\frac{-3\pm\sqrt{23}}{2}$$

Solve each of the quadratic equations below by **completing the square**. Remember that this may require you to multiply both sides of the equation first so that the coefficient on x^2 becomes one.

[1] $5x^2+20x-4=0$

[2] $3x^2+9x=5$

[3] $2x^2-16x=6$

[4] $4x^2+4x=3$

[5] $9x^2+18x-2=0$

[6] $6x^2+30x-5=2$

Mixed Practice

You now know how to solve a quadratic equation by: isolating the squared term, factoring then setting factors equal to zero, and completing the square. For the set of quadratic equations below, choose the method you believe will work best then solve the equation.

[7] $7x^2-6=12x^2-56$

[8] $9x^2-15=6x^2+66$

[9] $162=90+2(x-4)^2$

[10] $100-3(2x-5)^2=40$

[11] $-3x(5x-4)=0$

[12] $2x(x-12)=0$

[13] $(2x+9)(x-8)=0$

[14] $(3x-4)(x+7)=0$

[15] $x^2+24=10x$

[16] $x^2-4x=77$

[17] $x^2+9x-7=0$

[18] $x^2-8x-2=0$

[19] $3x^2-2x=4$

[20] $2x^2+6x=5$