The Discriminant

UNIT 7A: QUADRATIC EQUATIONS | PROBLEM SET 7A 06

The solutions to the quadratic equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

Notice that there appears a $\pm$ sign in the formula. If we use this sign to separate the formula into two different rational expressions we obtain the equivalent formula $x = -\frac{b}{2a} + \frac{\sqrt{b^2-4ac}}{2a} \;\; or \;\; -\frac{b}{2a} - \frac{\sqrt{b^2-4ac}}{2a}$.

The expression $b^2 - 4ac$ is called the **discriminant** of the quadratic formula because it determines the nature of the solutions to the equation. Complete the statements below to describe the nature of the solutions determined by the discriminant under four different circumstance.

[1] $x = -\frac{b}{2a} + \frac{\sqrt{positive\ perfect\ square}}{2a} \;\; or \;\; -\frac{b}{2a} - \frac{\sqrt{positive\ perfect\ square}}{2a}$

If the discriminant is a positive perfect square then the quadratic equation has ________________

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[2] $x = -\frac{b}{2a} + \frac{\sqrt{positive\ but\ not\ a\ perfect\ square}}{2a} \;\; or \;\; -\frac{b}{2a} - \frac{\sqrt{positive\ but\ not\ a\ perfect\ square}}{2a}$

If the discriminant is positive but is not a perfect square then the quadratic equation has ________

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[3] $x = -\frac{b}{2a} + \frac{\sqrt{zero}}{2a} \;\; or \;\; -\frac{b}{2a} - \frac{\sqrt{zero}}{2a}$

If the discriminant is zero then the quadratic equation has __________________________________

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[4] $x = -\frac{b}{2a} + \frac{\sqrt{negative}}{2a} \;\; or \;\; -\frac{b}{2a} - \frac{\sqrt{negative}}{2a}$

If the discriminant is negative then the quadratic equation has ______________________________

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Consider this set of quadratic equations. Calculate the discriminant of each quadratic equation in order to determine the nature of the solutions to the equation. Then rewrite the equation in the appropriate category below.

[ex] $2x(x-4)=7 \rightarrow 2x^2-8x-7=0 \rightarrow a=2, b=-8, c=-7$

$discriminant = b^2-4ac = (-8)^2-4(2)(-7)=64+56=120$

The discriminant is positive but is not a perfect square. Therefore the equation has two different and irrational number solutions.

[1] $x^2+3x-9=0$

[2] $x^2-4x-5=0$

[3] $3x^2-3x+5=0$

[4] $9x^2-6x+1=0$

[5] $\frac{1}{2}x^2=\frac{3}{4}x-\frac{1}{3}$

[6] $(2x+7)(x-2)=6$

[7] $\frac{2x^2+6x}{3}=-\frac{3}{2}$

Two different real and rational number solutions	*Two different real and irrational number solutions* $2x(x-4)=7$
One rational number solution	*Two conjugate imaginary number solutions*